An application of generalized Morrey spaces to unique continuation property of the quasilinear elliptic equations

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We thank to the referee for the valuable suggestions.

Referee’s Report 1. 1. Page 5, Equation (4.2): I believe that the last assumption should be

\[ \xi A(x, u, \xi) \geq |\xi|^p - d(x)|u|^p, \]

otherwise it is unclear to me how to obtain the first inequality in (4.19).

2. Page 1, line 1 in the abstract: we study nonnegative weak solutions (“the” is unnecessary.)

3. Page 1, line 2 in the abstract: in a bounded open set.

4. Page 1, line 3 in the abstract: \( \delta \) an arbitrary positive number.

5. Page 1, line 7 from below: every nonnegative weak solution ... at a point in \( \Omega \) satisfies \( u = 0 \).

6. Page 1, line 5 from below: assuming \( V \) belongs to the Morrey space.


8. Page 2, line 9: then proved the inequality.

9. Page 2, line 11: assuming the potential belongs to the generalized Morrey space.

10. Page 2, line 18: they started their main proof.

11. Page 2, line 4 from below: many papers which discuss the inclusion.


13. Page 4, line 1 from below: on \( \partial \Omega \).

14. Page 5, line 7: whose extensions ... are such that.

15. Page 5, line 18: a strictly positive.

16. Page 5, line 9 from below: every nonnegative weak solution ... at some \( x_0 \in \Omega \) satisfies \( u \equiv 0 \).

17. Page 6, line 1: by a similar method.

18. Page 6, line 9 from below: there exist \( g, h \in L^p(\Omega) \).

19. Page 6, line 6 from below: a sequence whose term is defined by.

20. Page 8, line 1: \( \phi(x)^p \) by using (4.8). (Period is unnecessary.)

21. Page 8, line 9: a sequence whose term is defined by.

22. Page 9, line 7 from below: Let \( \epsilon > 0 \) be fixed later.

23. Page 10, line 3: We infer from (4.21) and (4.22) that.

24. Page 11, line 2 from below: We infer ... and (4.32) that.
responses to referee's report 1. we have revised:
1. "...and \( B = B(x, s, \xi) : \Omega \times \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R} \) are two continuous functions and satisfy:
\[
\begin{align*}
|A(x, u, \xi)| &\leq a|\xi|^{p-1} + b(x)|u|^{p-1}, \\
|B(x, u, \xi)| &\leq c(x)|\xi|^{p-1} + d(x)|u|^{p-1}, \\
\xi A(x, u, \xi) &\geq |\xi|^{p} - d(x)|u|^{p},
\end{align*}
\]
for almost all \( x \in \Omega \), for all...
2. "...we study nonnegative weak solutions..."
3. "...in a bounded open set..."
4. "...a nonnegative solution and \( \delta \) an arbitrary positive real number..."
5. "...every nonnegative weak solution \( u \) of \( Lu = 0 \) which vanishes with infinite order at a point in \( \Omega \) satisfies \( u = 0 \)..."
6. "...assuming \( V \) belongs to the Morrey space..."
7. "...[3] generalized these results..."
8. "...then proved the inequality assuming..."
9. "...assuming the potential belongs to the generalized Morrey space..."
10. "...they started their main proof..."
11. "...many papers which discuss the inclusion..."
12. "...known as the John-Nirenberg..."
13. "...on \( \partial \Omega \)"
14. "...whose extensions with zero value outside of \( \Omega \) are such that..."
15. "...a strictly positive..."
16. "...vanishes with infinite order at some \( x_0 \in \Omega \) satisfies \( u \equiv 0 \)..."
17. "...by a similar method..."
18. "...there exist \( g, h \in L^p(\Omega) \)..."
19. "...a sequence whose term is defined by..."
20. The period is deleted.
21. "...a sequence whose term is defined by..."
22. "Let \( \epsilon > 0 \) be fixed later..."
23. "We infer from (4.21) and (4.22) that..."
24. "We infer ... and (4.32) that ...
25. "... \( K_{14} = K_{14}(n) \) is the positive constant which appears..."