Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, x(x): xxx–xxx
DOI:
Received:
Accepted:
Published:

Type of article

Maps on C^*-algebras which are skew Lie triple derivations or homomorphisms at one point

Zhonghua Wang[1], **and Xiuhai Fei** [2,*]

[1] School of Science, Xi'an Shiyou University, Xi'an, P. R. China

[2] School of Mathematics and Physics, West Yunnan University, Lincang, P.R. China

* **Correspondence:** Email: xiuhaifei@snnu.edu.cn.

Abstract: In this paper, we show that every continuous linear map between unital C^*-algebras which is skew Lie triple derivable at the identity is a $*$-derivation and every continuous linear map between unital C^*-algebras which is a skew Lie triple homomorphism at the identity is a Jordan $*$-homomorphism.

Keywords: C^***-algebra, skew Lie triple derivation, Jordan derivation, skew Lie triple homomorphism**

1. Introduction and basic definitions

Homomorphisms and derivations are the most intensively studied classes of operators on Banach algebras or other algebras. Let A be an algebra and M be an A-bimodule, $d : A \to M$ be a linear map. If for any $x, y \in A$,

$$d(xy) = d(x)y + xd(y), \tag{1.1}$$

then d is called a derivation. If for any $x \in A$, $d(x^2) = d(x)x + xd(x)$, then d is called a Jordan derivation. And if for any $x, y \in A$, $d([x, y]) = [d(x), y] + [x, d(y)]$, where $[x, y] = xy - yx$ is the Lie product of x and y, then d is called a Lie derivation. Clearly, a derivation is a Jordan derivation and is a Lie derivation. But a Jordan or Lie derivation need not certainly be a derivation. The standard problem is to find conditions implying that a Jordan or a Lie derivation is actually a derivation. Along this line, there are fruitful results. See for example [1–7].

Recently, many studies are concerned with finding the standard form of linear maps satisfying the derivation type equation for special pairs of x and y. For example, [8–10] studied linear maps which satisfy the derivation type equation for any x, y with $xy = 0$ and [11–13]studied linear maps which satisfy the derivation type equation for any x, y with $xy = 1$.

A map φ from a $*$-algebra A into a bimodule M over A is called a skew Lie derivation if

$$\varphi([a,b]_*) = [\varphi(a), b]_* + [a, \varphi(b)]_* \tag{1.2}$$

for all $a, b \in A$, is called skew Lie derivable at the point $z \in A$ if (1.2) holds for all $a, b \in A$ with $ab = z$, where $[a,b]_* = ab - ba^*$. A linear map φ from A into another C^*-algebra B is called a skew Lie homomorphism at z if $\varphi([a,b]_*) = [\varphi(a), \varphi(b)]_*$ for all $a, b \in A$ with $ab = z$. If φ is a skew Lie homomorphism at all elements of A, then it is called a skew Lie homomorphism on A. For their special importance, these maps attracted many authors' attention in the past decades (see [14–17]).

In [18], Li, Zhao, and Chen introduced the concept of nonlinear skew Lie triple derivations and showed every nonlinear skew Lie triple derivation between factors is an additive $*$-derivation.

Difinition 1.1 An additive map $\varphi : A \to M$ is called a nonlinear skew Lie triple derivation if

$$\varphi([[a,b]_*, c]_*) = [[\varphi(a), b]_*, c]_* + [[a, \varphi(b)]_*, c]_* + [[a,b]_*, \varphi(c)]_* \tag{1.3}$$

for all a, b, c in A, and is called skew Lie triple derivable at the point $z \in A$ if Eq. (1.3) holds for all $a, b, c \in A$ with $ab = z$ and $c = a$.

For unital C^*-algebras A and B, in the present peper, we study two types of continuous maps. One of them is the type of continuous linear maps from A into B, which are skew Lie triple derivable at the identity. Another is the type of continuous linear maps from A into B which are skew Lie triple homomorphisms at the identity (see section 3 for more details).

2. Skew Lie triple derivations at the identity

In this section we will study linear maps between unital C^*-algebras which are skew Lie triple derivable at the identity. Throughout this section, B will be a unital C^*-algebra, A will be a C^*-subalgebra of B and $1_A = 1_B$, A_{sa} will be the set of all self-adjoint elements in A, $\varphi : A \to B$ will be a linear map. Firstly, let us explore the behavior of the identity 1 under φ when φ is skew Lie triple derivable at the identity.

Lemma 2.1. *If $\varphi : A \to B$ is skew Lie triple derivable at the identity, then $\varphi(1) = \varphi(1)^*$.*

Proof. Since $1 \cdot 1 = 1$ and $[[1,1]_*, 1]_* = 0$, we have

$$0 = [[\varphi(1), 1]_*, 1]_* + [[1, \varphi(1)]_*, 1]_* + [[1,1]_*, \varphi(1)]_* = \varphi(1) - \varphi(1)^* - (\varphi(1) - \varphi(1)^*)^*,$$

from which it follows that $\varphi(1) = \varphi(1)^*$.

Theorem 2.1. *Let $\varphi : A \to B$ be a continuous linear map which is skew Lie triple derivable at the identity, then it is a $*$-derivation.*

proof. (1) Firstly, we show φ is selfadjoint, i.e., $\varphi(x^*) = \varphi(x)^*$ for all $x \in A$. Put any $a \in A_{sa}$, e^{ita} is a unitary for each $t \in \mathbb{R}$ and $[[\mathrm{e}^{ita}, \mathrm{e}^{-ita}]_*, \mathrm{e}^{ita}]_* = \mathrm{e}^{3ita} - \mathrm{e}^{-ita}$. Thus we deduce that

$$\begin{aligned}
&\varphi(\mathrm{e}^{3ita}) - \varphi(\mathrm{e}^{-ita}) \\
=&[[\varphi(\mathrm{e}^{ita}), \mathrm{e}^{-ita}]_*, \mathrm{e}^{ita}]_* + [[\mathrm{e}^{ita}, \varphi(\mathrm{e}^{-ita})]_*, \mathrm{e}^{ita}]_* + [[\mathrm{e}^{ita}, \mathrm{e}^{-ita}]_*, \varphi(\mathrm{e}^{ita})] \\
=& -\mathrm{e}^{-ita}\varphi(\mathrm{e}^{ita})^*\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{ita})\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})\mathrm{e}^{ita} - \mathrm{e}^{ita}\varphi(\mathrm{e}^{ita})^*\mathrm{e}^{-ita} \\
& - \mathrm{e}^{-2ita}\varphi(\mathrm{e}^{ita}) + \varphi(\mathrm{e}^{ita})\mathrm{e}^{2ita} + \varphi(\mathrm{e}^{ita}) - \varphi(\mathrm{e}^{-ita}) + \mathrm{e}^{2ita}\varphi(\mathrm{e}^{-ita})^* - \mathrm{e}^{2ita}\varphi(\mathrm{e}^{ita})^*.
\end{aligned} \tag{2.1}$$

By taking derivative of Eq. (2.1) at t, we obtain that

$$\begin{aligned}
&\varphi(3a\mathrm{e}^{3ita}) + \varphi(a\mathrm{e}^{-ita})\\
=&a\mathrm{e}^{-ita}\varphi(\mathrm{e}^{ita})^*\mathrm{e}^{ita} + \mathrm{e}^{-ita}\varphi(a\mathrm{e}^{ita})^*\mathrm{e}^{ita} - \mathrm{e}^{-ita}\varphi(\mathrm{e}^{ita})^*a\mathrm{e}^{ita}\\
&+ a\mathrm{e}^{ita}\varphi(\mathrm{e}^{ita})\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(a\mathrm{e}^{ita})\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{ita})a\mathrm{e}^{ita}\\
&+ a\mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})\mathrm{e}^{ita} - \mathrm{e}^{ita}\varphi(a\mathrm{e}^{-ita})\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})a\mathrm{e}^{ita}\\
&- a\mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})^*\mathrm{e}^{-ita} - \mathrm{e}^{ita}\varphi(a\mathrm{e}^{-ita})^*\mathrm{e}^{-ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})^*a\mathrm{e}^{-ita}\\
&+ 2a\mathrm{e}^{-2ita}\varphi(\mathrm{e}^{ita}) - \mathrm{e}^{-2ita}\varphi(a\mathrm{e}^{ita}) + \varphi(a\mathrm{e}^{ita})\mathrm{e}^{2ita} + 2\varphi(\mathrm{e}^{ita})a\mathrm{e}^{2ita} + \varphi(a\mathrm{e}^{ita}) + \varphi(a\mathrm{e}^{-ita})\\
&+ 2a\mathrm{e}^{2ita}\varphi(\mathrm{e}^{-ita})^* + \mathrm{e}^{2ita}\varphi(a\mathrm{e}^{-ita})^* - 2a\mathrm{e}^{2ita}\varphi(\mathrm{e}^{ita})^* + \mathrm{e}^{2ita}\varphi(a\mathrm{e}^{ita})^*. \qquad (2.2)
\end{aligned}$$

Put $t = 0$ and $a = 1$ in Eq. (2.2), then we get $\varphi(1) = 0$. Again put $t = 0$ in Eq. (2.2), noted that $\varphi(1) = 0$, we get

$$\varphi(a) = \varphi(a)^*, \quad a \in A_{sa}.$$

For each $x \in A$, there are $a, b \in A_{sa}$ such that $x = a + ib$. Hence, $\varphi(x^*) = \varphi(a) - i\varphi(b) = \varphi(x)^*$.

(2) Now we show φ is a $*$-derivation. Taking derivative of Eq. (2.2) in $t = 0$ yields that

$$\varphi(a^2) = \varphi(a)a + a\varphi(a). \qquad (2.3)$$

Put any $a, b \in A_{sa}$, then

$$\varphi((a + b)^2) = \varphi(a + b)(a + b) + (a + b)\varphi(a + b).$$

So

$$\varphi(ab + ba) = \varphi(a)b + a\varphi(b) + \varphi(b)a + b\varphi(a). \qquad (2.4)$$

For any $x \in A$, there are $a, b \in A$ such that $x = a + ib$. Hence,

$$\begin{aligned}
\varphi(x^2) =&\varphi((a^2 - b^2) + i(ab + ba))\\
=&(\varphi(a)a + a\varphi(a) - \varphi(b)b - b\varphi(b)) + i(\varphi(a)b + a\varphi(b) + \varphi(b)a + b\varphi(a))\\
=&\varphi(x)x + x\varphi(x).
\end{aligned}$$

Therefore, φ is a Jordan derivation. By [4, Theorem 6.3], φ is a $*$-derivation.

3. Skew Lie triple homomorphisms at the identity

Let A, B be unital C^*-algebras, $\varphi : A \to B$ a linear map. If

$$\varphi([[x, y]_*, x]_*) = [[\varphi(x), \varphi(y)]_*, \varphi(x)]_*$$

for all $x, y \in A$ with $xy = 1$, then φ is called a skew Lie triple homomorphism at the identity. Recall that a Jordan $*$-homomorphism between C^*-algebras is a linear map φ such that $\varphi(x^2) = \varphi(x)^2$ and $\varphi(x^*) = \varphi(x)^*$. In this section we prove that every linear continuous skew Lie triple homomorphism at the identity is a Jordan $*$-homomorphism. Throughout this section A and B will be unital C^*-algebras.

Lemma 3.1. *Let $\varphi : A \to B$ be a linear continuous skew Lie triple homomorphism at the identity. Then $\varphi(1)$ is a partial isometry.*

Proof. Since the product of 1 and itself is 1 and $[[1,1]_*,1]_*=0$, then

$$0=[[\varphi(1),\varphi(1)]_*,\varphi(1)]_*=\varphi(1)^3-\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)\varphi(1)^*\varphi(1)^*+\varphi(1)\varphi(1)\varphi(1)^*.$$

Hence,

$$\varphi(1)^3-\varphi(1)\varphi(1)^*\varphi(1)^*=\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)\varphi(1)\varphi(1)^*. \tag{3.1}$$

For any $a\in A_{sa}$, e^{ita} is a unitary for each real number t and $[[\mathrm{e}^{ita},\mathrm{e}^{-ita}]_*,\mathrm{e}^{ita}]_*=\mathrm{e}^{3ita}-\mathrm{e}^{-ita}$. Thus we deduce that

$$\begin{aligned}\varphi(\mathrm{e}^{3ita})-\varphi(\mathrm{e}^{-ita})=&[[\varphi(\mathrm{e}^{ita}),\varphi(\mathrm{e}^{-ita})]_*,\varphi(\mathrm{e}^{ita})]_*\\ =&\varphi(\mathrm{e}^{ita})\varphi(\mathrm{e}^{-ita})\varphi(\mathrm{e}^{ita})-\varphi(\mathrm{e}^{-ita})\varphi(\mathrm{e}^{ita})^*\varphi(\mathrm{e}^{ita})\\ &-\varphi(\mathrm{e}^{ita})\varphi(\mathrm{e}^{-ita})^*\varphi(\mathrm{e}^{ita})^*+\varphi(\mathrm{e}^{ita})\varphi(\mathrm{e}^{ita})\varphi(\mathrm{e}^{-ita})^*.\end{aligned}$$

Take derivative at t, then

$$\begin{aligned}3\varphi(a\mathrm{e}^{3ita})+\varphi(a\mathrm{e}^{-ita})=&\varphi(a\mathrm{e}^{ita})\varphi(\mathrm{e}^{-ita})\varphi(\mathrm{e}^{ita})-\varphi(\mathrm{e}^{ita})\varphi(a\mathrm{e}^{-ita})\varphi(\mathrm{e}^{ita})\\ &+\varphi(\mathrm{e}^{ita})\varphi(\mathrm{e}^{-ita})\varphi(a\mathrm{e}^{ita})+\varphi(a\mathrm{e}^{-ita})\varphi(\mathrm{e}^{ita})^*\varphi(\mathrm{e}^{ita})\\ &+\varphi(\mathrm{e}^{-ita})\varphi(a\mathrm{e}^{ita})^*\varphi(\mathrm{e}^{ita})-\varphi(\mathrm{e}^{-ita})\varphi(\mathrm{e}^{ita})^*\varphi(a\mathrm{e}^{ita})\\ &-\varphi(a\mathrm{e}^{ita})\varphi(\mathrm{e}^{-ita})^*\varphi(\mathrm{e}^{ita})^*-\varphi(\mathrm{e}^{ita})\varphi(a\mathrm{e}^{-ita})^*\varphi(\mathrm{e}^{ita})^*\\ &+\varphi(\mathrm{e}^{ita})\varphi(\mathrm{e}^{-ita})^*\varphi(a\mathrm{e}^{ita})^*+\varphi(a\mathrm{e}^{ita})\varphi(\mathrm{e}^{ita})\varphi(\mathrm{e}^{-ita})^*\\ &+\varphi(\mathrm{e}^{ita})\varphi(a\mathrm{e}^{ita})\varphi(\mathrm{e}^{-ita})^*+\varphi(\mathrm{e}^{ita})\varphi(\mathrm{e}^{ita})\varphi(a\mathrm{e}^{-ita})^*.\end{aligned} \tag{3.2}$$

Put $t=0$ and $a=1$, then

$$4\varphi(1)=\varphi(1)^3+\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)\varphi(1)^*\varphi(1)^*+3\varphi(1)\varphi(1)\varphi(1)^*. \tag{3.3}$$

By taking derivative of Eq. (3.2) at $t=0$, we get

$$\begin{aligned}-8\varphi(a^2)=&-\varphi(a^2)\varphi(1)^2+\varphi(1)\varphi(a^2)^*\varphi(1)+\varphi(1)\varphi(1)^*\varphi(a^2)^*-\varphi(1)\varphi(a^2)\varphi(1)^*\\ &-\varphi(1)\varphi(a^2)\varphi(1)+\varphi(a^2)\varphi(1)^*\varphi(1)+\varphi(1)\varphi(a^2)^*\varphi(1)^*-\varphi(1)^2\varphi(a^2)^*\\ &-\varphi(1)^2\varphi(a^2)+\varphi(1)\varphi(1)^*\varphi(a^2)+\varphi(a^2)[\varphi(1)^*]^2-\varphi(a^2)\varphi(1)\varphi(1)^*\\ &+2[\varphi(a)^2\varphi(1)+\varphi(a)\varphi(a)^*\varphi(1)-\varphi(1)[\varphi(a)^*]^2-\varphi(1)\varphi(a)\varphi(a)^*]\\ &-2[\varphi(a)\varphi(1)\varphi(a)+\varphi(1)\varphi(a)^*\varphi(a)+\varphi(a)\varphi(1)^*\varphi(a)^*+\varphi(a)^2\varphi(1)^*]\\ &+2[\varphi(1)\varphi(a)^2-\varphi(a)\varphi(1)^*\varphi(a)+\varphi(a)\varphi(a)^*\varphi(1)^*-\varphi(a)\varphi(1)\varphi(a)^*].\end{aligned} \tag{3.4}$$

By putting $a=1$ in Eq. (3.4), we get

$$8\varphi(1)=\varphi(1)^3-\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)\varphi(1)^*\varphi(1)^*+9\varphi(1)\varphi(1)\varphi(1)^*. \tag{3.5}$$

Multiplying Eq. (3.3) by 2 and subtracting Eq. (3.5) yield that

$$0=\varphi(1)^3+3\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)(\varphi(1)^*)^2-3\varphi(1)^2\varphi(1)^*,$$

which implies

$$\varphi(1)^3-\varphi(1)[\varphi(1)^*]^2=3\varphi(1)^2\varphi(1)^*-3\varphi(1)\varphi(1)^*\varphi(1). \tag{3.6}$$

It follows from Eq. (3.1) and Eq. (3.6) that

$$\varphi(1)^3 = \varphi(1)[\varphi(1)^*]^2, \quad \varphi(1)^2\varphi(1)^* = \varphi(1)\varphi(1)^*\varphi(1).$$

Now combine the above two equations and Eq. (3.3), then we get $\varphi(1) = \varphi(1)\varphi(1)^*\varphi(1)$. Hence, $\varphi(1)$ is a partial isometry.

If furthermore $\varphi(1) = 1$, then we can show the following main theorem.

Theorem 3.2. *Let $\varphi : A \to B$ be a linear continuous skew Lie triple homomorphism at the identity. If $\varphi(1) = 1$, then φ is a Jordan $*$-homomorphism.*

Proof. (1) Since $\varphi(1) = 1$, by putting $t = 0$ in Eq. (3.2), we obtain that

$$3\varphi(a) + \varphi(a) = \varphi(a) - \varphi(a) + \varphi(a) + \varphi(a) + \varphi(a)^* - \varphi(a) - \varphi(a) - \varphi(a)^* + \varphi(a)^* + \varphi(a) + \varphi(a) + \varphi(a)^*,$$

i.e.,$\varphi(a) = \varphi(a)^*$. As in the proof of Theorem 2.2., we can see that $\varphi(x^*) = \varphi(x)^*$ for all $x \in A$.

(2) Since $\varphi(1) = 1$ and $\varphi(a) = \varphi(a)^*$, it follows from Eq. (3.4) that

$$\varphi(a^2) = \varphi(a)^2, \quad a \in A_{sa}.$$

Replacing a by $a + b$ for $a, b \in A_{sa}$, we get

$$\varphi(ab + ba) = \varphi(a)\varphi(b) + \varphi(b)\varphi(a), \quad a, b \in A_{sa}.$$

Now for each $x \in A$, there are $a, b \in A_{sa}$ such that $x = a + ib$. So

$$\varphi(x^2) = \varphi(a^2 - b^2 + i(ab + ba)) = \varphi(a)^2 - \varphi(b)^2 + i(\varphi(a)\varphi(b) + \varphi(b)\varphi(a)) = \varphi(x)^2,$$

and so φ is a Jordan $*$-homomorphism.

For any partial isometry e in a C^*-algebra A, e^*e and ee^* are projections. A can be decomposed as a direct sum of the form

$$A = ee^*Ae^*e \oplus (1 - ee^*)Ae^*e \oplus ee^*A(1 - e^*e) \oplus (1 - ee^*)A(1 - e^*e).$$

From Lemma 3.1., it follows that $\varphi(1)$ is a partial isometry. Let $\varphi(1)\varphi(1)^* = p$ and $\varphi(1)^*\varphi(1) = q$, then we can decompose B as

$$B = pBq \oplus p^\perp Bq \oplus pBq^\perp \oplus p^\perp Bq^\perp,$$

where $p^\perp = 1 - p$ and $q^\perp = 1 - q$. Let $B_0(\varphi(1)) = pBq, B_2(\varphi(1)) = p^\perp Bq^\perp$, we can show the following corollary.

Corollary 3.1. *Let $\varphi : A \to B$ be a linear continuous skew Lie triple homomorphism at the identity. If $\varphi(1)^* = \varphi(1)$, then $\varphi(a) = \varphi(a)^*$ for all $a \in A_{sa}$ and $\varphi(A) \subset B_0(\varphi(1))$.*

Proof. Since $\varphi(1)^* = \varphi(1)$, $p = q = \varphi(1)^2$. By putting $t = 0$ in Eq. (3.2), we obtain that

$$\begin{aligned}4\varphi(a) =& \varphi(a)\varphi(1)\varphi(1) - \varphi(1)\varphi(a)\varphi(1) + \varphi(1)\varphi(1)\varphi(a) + \varphi(a)\varphi(1)^*\varphi(1) \\ &+ \varphi(1)\varphi(a)^*\varphi(1) - \varphi(1)\varphi(1)^*\varphi(a) - \varphi(a)\varphi(1)^*\varphi(1)^* - \varphi(1)\varphi(a)^*\varphi(1)^* \\ &+ \varphi(1)\varphi(1)^*\varphi(a)^* + \varphi(a)\varphi(1)\varphi(1)^* + \varphi(1)\varphi(a)\varphi(1)^* + \varphi(1)\varphi(1)\varphi(a)^* \\ =& 2\varphi(a)p + 2p\varphi(a)^*. \end{aligned} \tag{3.7}$$

Hence, $\varphi(a) = \varphi(a)^*$ for all $a \in A_{sa}$. It follows from Eq. (3.7) that

$$\varphi(a) = \frac{1}{2}\varphi(a)p + \frac{1}{2}p\varphi(a).$$

So $p^\perp\varphi(a)p^\perp = p\varphi(a)p^\perp = p^\perp\varphi(a)p = 0$ and so $\varphi(a) = p\varphi(a)p \in B_0(\varphi(1))$. Therefore, $\varphi(A) \subset B_0(\varphi(1))$.

Corollary 3.2. *Let $\varphi : A \to B$ be a linear continuous skew Lie triple homomorphism at the identity. Then φ is a Jordan $*$-homomorphism if and only if $\varphi(1)$ is a projection.*

Proof. If φ is a Jordan $*$-homomorphism, then $\varphi(1) = \varphi(1^2) = \varphi(1)^2$ and $\varphi(1)^* = \varphi(1)$. So $\varphi(1)$ is a projection.

Conversely, if $\varphi(1)$ is a projection, then $\varphi(1) = p$ is the identity of the subalgebra $B_0(\varphi(1))$. By Corollary 3.1., we can regard φ as a map from A into $B_0(\varphi(1))$. Hence, by Theorem 3.2, φ is a Jordan $*$-homomorphism.

It is not hard to see that the continuity of the linear map φ is very important in this paper. The automatical continuity of some maps on operator algebra is an important problem. See for example [19]. Let φ be a linear map which is skew Lie triple derivable at the identity or is a skew Lie triple homomorphism at the identity. It is natural to ask whether φ is automatically continuous.

Acknowledgments

This research is supported by the National Natural Science Foundation of China (No.11901451), Talent Project Foundation of Yunnan Provincial Science and Technology Department (No.202105AC160089), and Natural Science Foundation of Yunnan Province(No.202101BA070001198)

Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Supplementary (if necessary)