Research article

Characterizations of intra-regular LA-semihyperrings in terms of their hyperideals

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Abstract: The purpose of this article is to investigate the class of intra-regular LA-semihyperrings. Then, characterizations of intra-regular LA-semihyperrings by the properties of many types of their hyperideals are obtained. Moreover, we present a construction of LA-semihyperrings from ordered LA-semirings.

Keywords: LA-semihypergroup; LA-semihyperring; intra-regular LA-semihyperring

Mathematics Subject Classification: 20M17, 20N20, 16Y60

1. Introduction

The algebraic structure of left almost semigroups (for short, LA-semigroups), which is a generalization of commutative semigroups, was first introduced by Kazim and Naseeruddin [20] in 1972. An Abel-Grassmann groupoid (for short, AG-groupoid) is another name for it [34]. A non-associative and a non-commutative algebraic structure that lies midway between a groupoid and a commutative semigroup is known as an LA-semigroup. Regularities are interesting and important properties to examine in LA-semigroups. In 2010, Khan and Asif [22] characterized intra-regular LA-semigroups by the properties of their fuzzy ideals. Later, Abdullah et al. [1] discussed characterizations of regular LA-semigroups using interval valued \((\alpha,\beta)\)-fuzzy ideals. Also, Khan et al. [24] characterized right regular LA-semigroups using their fuzzy left ideals and fuzzy right ideals. In 2016, Khan et al. [26] characterized the class of \((m,n)\)-regular LA-semigroups by their \((m,n)\)-ideals. Some characterizations of weakly regular LA-semigroups by using the smallest ideals and fuzzy ideals of LA-semigroups are investigated by Yousafzai et al. [46]. In addition, Sezer [37] have used the concept of soft sets to characterize regular, intra-regular, completely regular, weakly regular and quasi-regular LA-semigroups. Now, many mathematicians have investigated various characterizations of LA-semigroups (see, e.g., [3, 9, 47]). Furthermore, some mathematicians have considered the notion of left almost semirings...
(for short, LA-semirings), that is a generalization of left almost rings (for short, LA-rings) [38], to have different features. In 2021, the left almost structures are now widely studied such as Elmoasry [14] studied the concepts of rough prime and rough fuzzy prime ideals in LA-semigroups, Massouros and Yaqoob [28] investigated the theory of left and right almost groups and focused on more general structures, and Rehman et al. [35] introduced the notion of neutrosophic LA-rings and discussed various types of ideals and establish several results to better understand the characteristic behavior of neutrosophic LA-rings. In addition, the concept of left almost has been investigated in various algebraic structures (for example, in ordered LA-semigroups [6, 19, 45], in ordered LA-Γ-semigroups [8], in gamma LA-rings and gamma LA-semigroups [25], in LA-polygroups [2, 41, 43]).

Marty [27] introduced the concept of hyperstructures, as a generalization of ordinary algebraic structures. The composition of two elements in an ordinary algebraic structure is an element, but in an algebraic hyperstructure, the composition of two elements is a nonempty set. Many authors have developed on the concept of hyperstructures (see, e.g., [4, 12, 39]). Rehman et al. [36] introduced the concept of left almost hypergroups (for short, LA-hypergroups) and gave the examples of LA-hypergroups. Moreover, they introduced the concept of LA-hyperrings and characterized LA-hyperrings by their hyperideals and hypersystems. Next, the concept of weak LA-hyperrings was investigated by Nawaz et al. [32]. In 2020, Hu et al. [18] extended the notion of neutrosophic to LA-hypergroups and strong pure LA-semihypergroups. The concept of left almost semihypergroups (for short, LA-semihypergroups) is a generalization of LA-semigroups and commutative semihypergroups developed by Hila and Dine [17]. An LA-semihypergroup is a non-associative and non-commutative hyperstructure midway between a hypergroupoid and a commutative semihypergroup. Yaqoob et al., [42] have characterized intra-regular LA-semihypergroups by using the properties of their left and right hyperideals. Then, Gulistan et al. [15] defined the class of regular LA-semihypergroups in terms of \((e \tau, e \Gamma \lor q \Delta)-\)cubic (resp., left, right, two-sided, bi, generalized bi, interior, quasi) hyperideals of LA-semihypergroups. Furthermore, Khan et al. [23] investigated some properties of fuzzy left hyperideals and fuzzy right hyperideals in regular and intra-regular LA-semihypergroups. Meanwhile, the notion of ordered LA-semihypergroups which is a generalization of LA-semihypergroups was introduced by Yaqoob and Gulistan [44]. Also, Azhar et al. discussed some results related with fuzzy hyperideals and generalized fuzzy hyperideals of ordered LA-semihypergroups [7, 16].

It is known that every semiring can be considered to be a semihyperring. This implies that some results in intra-regular semihyperrings generalized the results in intra-regular semirings. The class of intra-regular semihyperrings was investigated by Nakkhasen and Pibaljommee [31] in 2019. Afterward, Nawaz et al. [33] introduced the notion of left almost semihyperrings (for short, LA-semihyperrings), which is a generalization of LA-semirings. Recently, Nakkhasen [30] characterized some classes of regularities in LA-semihyperrings, that is, weakly regular LA-semihyperrings and regular LA-semihyperrings by the properties of their hyperideals. In this paper, we are interested in the class of intra-regular LA-semihyperrings. Then, we give some characterizations of intra-regular LA-semihyperrings by means of their hyperideals. In addition, we show how ordered LA-semirings can be used to create LA-semihyperrings.
2. Preliminaries

First, we will review some fundamental notions and properties that are needed for this study. Let $H$ be a nonempty set. Then, the mapping $\circ : H \times H \to \mathcal{P}(H)$ is called a hyperoperation (see, e.g., [10, 11, 40]) on $H$ where $\mathcal{P}(H) = \mathcal{P}(H) \setminus \{\emptyset\}$ denotes the set of all nonempty subsets of $H$. A hypergroupoid is a nonempty set $H$ together with a hyperoperation $\circ$ on $H$. If $x \in H$ and $A, B$ are two nonempty subsets of $H$, then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, A \circ x = A \circ \{x\} \text{ and } x \circ B = \{x\} \circ B.$$ 

A hypergroupoid $(H, \circ)$ is called an LA-semihypergroup [17] if for all $x, y, z \in H, (x \circ y) \circ z = (z \circ y) \circ x$. This law is known as a left invertive law. For any nonempty subsets $A, B$ and $C$ of an LA-semihypergroup $(H, \circ)$, we have that $(A \circ B) \circ C = (C \circ B) \circ A$.

A hyperstructure $(S, +, \cdot)$ is called an LA-semihyperring [33] if it satisfies the following conditions:

(i) $(S, +)$ is an LA-semihypergroup;

(ii) $(S, \cdot)$ is an LA-semihypergroup;

(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ and $(y + z) \cdot x = y \cdot x + z \cdot x$ for all $x, y, z \in S$.

Example 2.1. Let $\mathbb{Z}$ be the set of all integers. The hyperoperations $\ominus$ and $\odot$ on $\mathbb{Z}$ are defined by $x \ominus y = \{y - x\}$ and $x \odot y = \{xy\}$ for all $x, y \in \mathbb{Z}$, respectively. We have that $(\mathbb{Z}, \ominus, \odot)$ is an LA-semihyperrings.

Example 2.2. [36] Let $S = \{a, b, c\}$ be a set with the hyperoperations $+$ and $\cdot$ on $S$ defined as follows:

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<th>$a$</th>
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<tr>
<td>$a$</td>
<td>${a}$</td>
<td>${a, b, c}$</td>
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<td>$c$</td>
<td>${a, b, c}$</td>
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<td>${a, b, c}$</td>
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Then, $(S, +, \cdot)$ is an LA-semihyperring.

Throughout this paper, we say an LA-semihyperring $S$ instead of an LA-semihyperring $(S, +, \cdot)$ and we write $xy$ instead of $x \cdot y$ for any $x, y \in S$.

The concepts listed below will be considered in this research, as they occurred in [33]. For any LA-semihyperring $S$, the medial law $(xy)(zw) = (xz)(yw)$ holds for all $x, y, z, w \in S$. An element $e$ of an LA-semihyperring $S$ is called a left identity (resp., pure left identity) if for all $x \in S$, $x \cdot e = e$. We have that $S^2 = S$, for any LA-semihyperring $S$ with a left identity $e$. If an LA-semihyperring $S$ contains a pure left identity $e$, then it is unique. In an LA-semihyperring $S$ with a pure left identity $e$, the paramedial law $(xy)(zw) = (wy)(zx)$ holds for all $x, y, z, w \in S$. An element $a$ of an LA-semihyperring $S$ with a left identity (resp., pure left identity) $e$ is called a left invertible (resp., pure left invertible) if there exists $x \in S$ such that $e \cdot xa$ (resp., $e = xa$). An LA-semihyperring $S$ is called a left invertible (resp., pure left invertible) if every element of $S$ is a left invertible (resp., pure left invertible). We observe that if an element $e$ is a pure left identity of an LA-semihyperring $S$, then $e$ is also a left identity, but the converse is not true in general, see in [30].

Lemma 2.3. [33] If $S$ is an LA-semihyperring with a pure left identity $e$, then $x(yz) = y(zx)$ for all $x, y, z \in S$. 

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Let $S$ be an LA-semihyperring. Then, the following law holds \((AB)(CD) = (AC)(BD)\) for all nonempty subsets \(A, B, C, D\) of \(S\). If an LA-semihyperring \(S\) contains the pure left identity \(e\), then \((AB)(CD) = (DB)(CA)\) and \(A(BC) = B(AC)\) for every nonempty subsets \(A, B, C, D\) of \(S\).

Let \(S\) be an LA-semihyperring and a nonempty subset \(A\) of \(S\) such that \(A + A \subseteq A\). Then:

(i) \(A\) is called a left hyperideal [33] of \(S\) if \(SA \subseteq A\);
(ii) \(A\) is called a right hyperideal [33] of \(S\) if \(AS \subseteq A\);
(iii) \(A\) is called a hyperideal [33] of \(S\) if it is both a left and a right hyperideal of \(S\);
(iv) \(A\) is called a quasi-hyperideal [33] of \(S\) if \(SA \cap AS \subseteq A\);
(v) \(A\) is called a bi-hyperideal [33] of \(S\) if \(AA \subseteq A\) and \((AS)A \subseteq A\).

\section*{Example 2.4.}
Let \(S = \{a, b, c, d\}\). Define hyperoperations + and \(\cdot\) on \(S\) by the following tables:

\[
\begin{array}{c|cccc}
+ & a & b & c & d \\
\hline
a & \{a\} & \{a, b\} & \{c\} & \{d\} \\
b & \{a, b\} & \{a, b\} & \{c\} & \{d\} \\
c & \{c\} & \{c\} & \{c\} & \{d\} \\
d & \{d\} & \{d\} & \{d\} & \{d\}
\end{array}
\quad
\begin{array}{c|cccc}
\cdot & a & b & c & d \\
\hline
a & \{a\} & \{a\} & \{a\} & \{a\} \\
b & \{a\} & \{a\} & \{a\} & \{a\} \\
c & \{a, a\} & \{a, a\} & \{a, a\} & \{a, b\} \\
d & \{a, a\} & \{a, b\} & \{a, b\} & \{a, b\}
\end{array}
\]

We can see that \((S, +, \cdot)\) is an LA-semihyperring. Consider \(A = \{a, b, c\}\) and \(B = \{a, c\}\). It is easy to see that \(A\) is a quasi-hyperideal of \(S\). In addition, \(B\) is a bi-hyperideal of \(S\), but it is not a quasi-hyperideal of \(S\) because \(SB \cap BS = \{a, b\} \not\subseteq B\).

A nonempty subset \(G\) of an LA-semihyperring \(S\) is called a generalized bi-hyperideal of \(S\) if \(G + G \subseteq G\) and \((GS)G \subseteq G\). Obviously, every bi-hyperideal of an LA-semihyperring \(S\) is a generalized bi-hyperideal, but the converse is not true in general. We can show this with the following example.

\section*{Example 2.5.}
From Example 2.4, consider \(G = \{a, c, d\}\). It is not difficult to show that \(G\) is a generalized bi-hyperideal of \(S\). But \(G\) is not a bi-hyperideal of \(S\), because \(c \cdot d = \{a, b\} \not\subseteq G\).

An ordered LA-semiring is a system \((S, +, \cdot, \leq)\) consisting of a nonempty set \(S\) such that \((S, +, \cdot)\) is an LA-semiring, \((S, \leq)\) is a partially ordered set, and for every \(a, b, x \in S\) the following conditions are satisfied: (i) if \(a \leq b\), then \(a + x \leq b + x\) and \(x + a \leq x + b\); (ii) if \(a \leq b\), then \(a \cdot x \leq b \cdot x\) and \(x \cdot a \leq x \cdot b\).

For an ordered LA-semiring \((S, +, \cdot, \leq)\) and \(x \in S\), we denote \((\cdot) = \{s \in S \mid s \leq x\}\).

In 2014, Amjad and Yousefzai [5] have shown that every ordered LA-semigroup \((S, \cdot, \leq)\) can be considered as an LA-semihypergroup \((S, \circ)\) where a hyperoperation \(\circ\) on \(S\) defined by

\[
a \circ b = \{x \in S \mid x \leq a \cdot b\} = (a \cdot b)\]

for all \(a, b \in S\).

Now, we apply this idea to construct an LA-semihyperring from an ordered LA-semiring as the following lemma.

\section*{Lemma 2.6.}
Let \((S, +, \cdot, \leq)\) be an ordered LA-semiring. Then \((S, \oplus, \odot)\) is an LA-semihyperring where the hyperoperations \(\oplus\) and \(\odot\) on \(S\) are defined by letting \(a, b \in S\),

\[
a \oplus b = \{x \in S \mid x \leq a + b\} = (a + b)\quad\text{and}\quad a \odot b = \{x \in S \mid x \leq a \cdot b\} = (a \cdot b).
\]

\textbf{Proof.}\ By the Example in [5], it follows that \((S, \oplus)\) and \((S, \odot)\) are LA-semihypergroups. Next, we will show that the hyperoperation \(\odot\) is distributive with respect to the hyperoperation \(\oplus\) on \(S\). First, we
claim that \( a \odot (b \oplus c) = (a \cdot (b + c)) \). Let \( t \in a \odot (b \oplus c) \). Then, \( t \in a \odot x \) for some \( x \in b \oplus c \). So, \( t \leq a \cdot x \leq a \cdot (b + c) \), then \( t \in (a \cdot (b + c)) \). Hence, \( a \odot (b \oplus c) \subseteq (a \cdot (b + c)) \). Let \( s \in (a \cdot (b + c)) \). Then, \( s \leq a \cdot (b + c) \), and so

\[
s \in a \odot (b + c) \subseteq \bigcup_{x \in b \oplus c} a \odot x = a \odot (b \oplus c).
\]

That is, \((a \cdot (b + c)) \subseteq a \odot (b \oplus c)\). It follows that \( a \odot (b \oplus c) = (a \cdot (b + c)) \). Next, we show that \((a \odot b) \oplus (a \odot c) = (a \cdot b + a \cdot c)\). Let \( t \in (a \odot b) \oplus (a \odot c) \). Then \( t \in x \oplus y \) for some \( x \in a \odot b \) and \( y \in a \odot c \). This implies that \( t \leq x + y \leq a \cdot b + a \cdot c \). Thus, \( t \in (a \cdot b + a \cdot c) \). Hence, \((a \odot b) \oplus (a \odot c) \subseteq (a \cdot b + a \cdot c)\). Let \( s \in (a \cdot b + a \cdot c) \). Then

\[
s \in a \cdot b + a \cdot c \subseteq \bigcup_{x \in a \odot b, y \in a \odot c} x \oplus y = (a \odot b) \oplus (a \odot c).
\]

Hence, \((a \cdot b + a \cdot c) \subseteq (a \odot b) \oplus (a \odot c)\). Therefore, \((a \odot b) \oplus (a \odot c) = (a \cdot b + a \cdot c)\). Since \((a \cdot (b + c)) = (a \cdot b + a \cdot c)\), we obtain that \( a \odot (b \oplus c) = (a \odot b) \oplus (a \odot c)\). Similarly, we can show that \((b \oplus c) \odot a = (b \odot a) \oplus (c \odot a)\). Consequently, \((S, \oplus, \odot)\) is an \(LA\)-semihyperring.

**Example 2.7.** Let \( S = \{a, b, c\} \) be a set with two binary operations \(+\) and \(\cdot\) on \( S \) defined as follows:

\[
\begin{array}{ccc}
+ & a & b & c \\
\hline
a & a & a & a \\
b & a & a & c \\
c & a & a & a \\
\end{array}
\quad
\begin{array}{ccc}
\cdot & a & b & c \\
\hline
a & a & a & a \\
b & a & a & c \\
c & a & a & a \\
\end{array}
\]

Then, \((S, +, \cdot)\) is an \(LA\)-semiring \([29]\). We define an order relation \(\leq\) on \( S \) by

\[
\leq = \{(a, a), (b, b), (c, c), (a, b), (a, c)\}.
\]

The figure of \(\leq\) on \( S \) is given by

\[
\begin{tikzpicture}
\node (a) at (0, 0) {a};
\node (b) at (1, 1) {b};
\node (c) at (2, 1) {c};
\draw (a) -- (b);
\draw (a) -- (c);
\end{tikzpicture}
\]

It is a routine matter to check that \((S, +, \cdot, \leq)\) is an ordered \(LA\)-semiring. We obtain that its associated \(LA\)-semihyperring \((S, \oplus, \odot)\) where \(\oplus\) and \(\odot\) are defined by Lemma 2.6 as follows:

\[
\begin{array}{ccc}
\oplus & a & b & c \\
\hline
a & \{a\} & \{a\} & \{a\} \\
b & \{a\} & \{a, c\} & \{b\} \\
c & \{a\} & \{a\} & \{c\} \\
\end{array}
\quad
\begin{array}{ccc}
\odot & a & b & c \\
\hline
a & \{a\} & \{a\} & \{a\} \\
b & \{a\} & \{a\} & \{a, c\} \\
c & \{a\} & \{a\} & \{a\} \\
\end{array}
\]

Now, we can see that \( A = \{a, b\} \) is a left hyperideal of \( S \), but it is not a right hyperideal of \( S \) because \( b \odot c = \{a, c\} \not\subseteq A \).

**Lemma 2.8.** \([30]\) Let \( S \) be an \(LA\)-semihyperring with a pure left identity \( e \). Then every right hyperideal of \( S \) is a hyperideal of \( S \).
**Lemma 2.9.** [30] Every left (resp., right) hyperideal of an LA-semihyperring $S$ is a quasi-hyperideal of $S$.

**Lemma 2.10.** Every left (resp., right) hyperideal of an LA-semihyperring $S$ is a bi-hyperideal of $S$.

*Proof.* Let $B$ be a left hyperideal of an LA-semihyperring $S$. Then, $BB \subseteq SB \subseteq B$, and so $(BS)B \subseteq SB \subseteq B$. Thus, $B$ is a bi-hyperideal of $S$. For the case right hyperideals, we can prove similarly. □

**Lemma 2.11.** [30] Let $S$ be an LA-semihyperring with a left identity $e$ such that $(xe)S \subseteq xS$ for all $x \in S$. Then every quasi-hyperideal of $S$ is a bi-hyperideal of $S$.

**Lemma 2.12.** [30] If $S$ is an LA-semihyperring with a pure left identity $e$, then for every $a \in S$, $a^2S$ is a hyperideal of $S$ such that $a^2 \subseteq a^2S$.

**Lemma 2.13.** If $S$ is an LA-semihyperring with a left identity $e$, then for every $a \in S$, $Sa$ is a left hyperideal of $S$ such that $a \in Sa$.

*Proof.* Assume that $S$ is an LA-semihypering with a left identity $e$. Let $a \in S$. Then, $a \in ea \subseteq Sa$ and $Sa + Sa = (S + S)a \subseteq Sa$. Now, by using paramedial law and left invertive law, we have

$$S(Sa) \subseteq (eS)(Sa) = (aS)(Se) = ((Se)S)a \subseteq Sa.$$  

It follows that $Sa$ is a left hyperideal of $S$. □

Let $J$ be a finite nonempty subset of $\mathbb{N}$ such that $J = \{j_1, j_2, j_3, \ldots, j_n\}$, where $j_1, j_2, j_3, \ldots, j_n \in \mathbb{N}$. For any $a \in S$, we denote

$$\sum_{i \in I} a_i = (\cdots ((a_{j_i} + a_{j_{i+1}}) + a_{j_{i+2}}) + \cdots) + a_{j_n}.$$

For any nonempty subsets $A$ and $B$ of LA-semihyperring $S$ and $a \in S$, we denote

$$\Sigma A = \{t \in S \mid t \in \sum_{i \in I} a_i, a_i \in A \text{ and } I \text{ is a finite nonempty subset of } \mathbb{N}\},$$

$$\Sigma AB = \{t \in S \mid t \in \sum_{i \in I} a_ib_i, a_i \in A, b_i \in B \text{ and } I \text{ is a finite nonempty subset of } \mathbb{N}\},$$

$$\Sigma a = \Sigma \{a\}.$$

**Remark 1.** Let $A$ and $B$ be any nonempty subsets of an LA-semihyperring $S$. Then the following statements hold:

(i) $A \subseteq \Sigma A$;

(ii) $A(\Sigma B) \subseteq \Sigma AB$ and $(\Sigma A)B \subseteq \Sigma AB$.

**Lemma 2.14.** Let $A$ be any nonempty subset of an LA-semihyperring $S$. If $A + A \subseteq A$, then $\Sigma A = aA$ and $\Sigma a = aA$ for all $a \in S$.  

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3. Characterizations of intra-regular LA-semihyperrings

In this section, we apply the concept of intra-regular LA-rings, defined in [21], to define the notion of intra-regular LA-semihyperrings and study some of its properties. Finally, we give some characterizations of intra-regular LA-semihyperrings by the properties of many types of hyperideals of LA-semihyperrings.

Definition 3.1. An LA-semihyperring $S$ is said to be intra-regular if for every $a \in S$, $a \in \Sigma(Sa^2)S$.

Example 3.2. Let $S = \{a, b, c\}$ be a set with the hyperoperations $+$ and $\cdot$ on $S$ defined as follows:

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<td>$a$</td>
<td>${a}$</td>
<td>${a, b, c}$</td>
<td>${a, b, c}$</td>
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<td>$b$</td>
<td>${b, c}$</td>
<td>${b, c}$</td>
<td>${b, c}$</td>
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<td>$c$</td>
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<tr>
<td>$b$</td>
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<td>${a, b, c}$</td>
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<tr>
<td>$c$</td>
<td>${a}$</td>
<td>${a, b, c}$</td>
<td>${a, b, c}$</td>
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Then, $(S, +, \cdot)$ is an LA-semihyperring [33]. Now, we can see that $S$ is intra-regular.

However, the set $S = \{a, b, c, d, e\}$ with two hyperoperations $\oplus$ and $\odot$ on $S$ as defined in Example 2.7 is not intra-regular, because $b \notin \{a\} = \Sigma(S \odot b^2) \odot S$.

Proposition 3.3. Every left (resp., right) hyperideal of an intra-regular LA-semihyperring $S$ is a hyperideal of $S$.

Proof. Let $S$ be an intra-regular LA-semihyperring and $x \in S$. Assume that $L$ is a left hyperideal of $S$ and $a \in L$. Then, $a \in \Sigma(Sa^2)S$. Now, by using Remark 1 and left invertive law, we have

$$ax \subseteq (\Sigma(Sa^2)S)x \subseteq \Sigma((Sa^2)S)x = \Sigma(xS)(Sa^2) \subseteq \Sigma SL \subseteq SL \subseteq L.$$

Thus, $L$ is a right hyperideal of $S$, and so $L$ is a hyperideal of $S$. Suppose that $R$ is a right hyperideal of $S$ and $r \in R$. Then,

$$xr \subseteq (\Sigma(Sx^2)S)r \subseteq \Sigma((Sx^2)S)r = \Sigma(rsS)(Sx^2) \subseteq \Sigma RS \subseteq \Sigma R \subseteq R.$$

Hence, $R$ is a left hyperideal of $S$. It follows that $R$ is a hyperideal of $S$.

Proposition 3.4. If $S$ is an intra-regular LA-semihyperring with a pure left identity $e$, then $\Sigma I^2 = I$ for every left hyperideal $I$ of $S$.

Proof. Assume that $S$ is an intra-regular LA-semihyperring with a pure left identity $e$. Let $I$ be a left hyperideal of $S$. Then, $\Sigma I^2 \subseteq I$. Let $a \in I$. By using left invertive law, medial law and Lemma 2.3, we have

$$a \in \Sigma(Sa^2)S = \Sigma(sa)S = \Sigma(s(aS))S = \Sigma(aS)(eS) = \Sigma(aS)S \subseteq \Sigma(Sa)((sa)e) \subseteq \Sigma(SI)(SI) \subseteq \Sigma II = \Sigma I^2.$$

Thus, $I \subseteq \Sigma I^2$. Therefore, $\Sigma I^2 = I$.

A (resp., left, right) hyperideal $P$ of an LA-semihyperring $S$ is called semiprime if for any $a \in S$, $a^2 \subseteq P$ implies $a \in P$. 

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**Proposition 3.5.** Every hyperideal of an intra-regular LA-semihyperring is semiprime.

**Proof.** Assume that $S$ is an intra-regular LA-semihyperring. Let $I$ be a hyperideal of $S$ and $a \in S$ such that $a^2 \subseteq I$. Then, $a \in \Sigma(Sa^2)S \subseteq \Sigma(SI)S \subseteq \Sigma I = I$. Hence, $I$ is semiprime. □

**Proposition 3.6.** Let $S$ be an LA-semihyperring $S$ with a pure left identity $e$. If $S$ satisfies $L \cup R = \Sigma LR$, for every left hyperideal $L$ and every right hyperideal $R$ of $S$ such that $R$ is semiprime, then $S$ is intra-regular.

**Proof.** Let $a \in S$. By Lemma 2.13 and Lemma 2.12, we have that $Sa$ is a left hyperideal and $a^2S$ is a right hyperideal of $S$ such that $a \in Sa$ and $a^2 \subseteq a^2S$, respectively. Thus, by the given assumption, $a \in a^2S$. Now, by using left invertive law, medial law and Lemma 2.3, we have

$$a \in Sa \cup a^2S = \Sigma(Sa)(a^2S) = \Sigma(Sa)((aa)S) \subseteq \Sigma(Sa)((aS)S) = \Sigma(aS)((Sa)S)$$

$$= \Sigma(a(Sa))(SS) = \Sigma(a(Sa))S = \Sigma(S(aa))S = \Sigma(Sa^2)S.$$  

This shows that $S$ is intra-regular. □

Next, we give characterizations of intra-regular LA-semihyperrings by means of (resp., left, right) hyperideals, quasi-hyperideals, bi-hyperideals and generalized bi-hyperideals of LA-semihyperrings as show by the following theorems.

**Theorem 3.7.** Let $S$ be an LA-semihyperring with a pure left identity $e$. Then $S$ is intra-regular if and only if $L = L^3$, for every left hyperideal $L$ of $S$.

**Proof.** Assume that $S$ is intra-regular. Let $L$ be any left hyperideal of $S$. Then, $L^3 = (LL)L \subseteq (SL)L \subseteq LL \subseteq L$. Now, let $a \in L$. By Lemma 2.12, $a^2S$ is a hyperideal of $S$ such that $a^2 \subseteq a^2S$. Thus, by given assumption and Proposition 3.5, we have that $a^2S$ is semiprime, and so $a \in a^2S$. Thus, by using left invertive law and Lemma 2.3, we have

$$a \in a^2S = (aa)S = (S(a)a \subseteq (S(a^2S))a = (a^2(S^2S))a = ((aa)S)a$$

$$= ((S(a)a)a \subseteq ((SL)L)L \subseteq (LL)L = L^3.$$  

Hence, $L \subseteq L^3$. Therefore, $L = L^3$.

Conversely, assume that $L = L^3$, for every left hyperideal $L$ of $S$. Let $a \in S$. By Lemma 2.13, $Sa$ is a left hyperideal of $S$ such that $a \in Sa$. Then, by given assumption and using medial law, we have

$$a \in Sa = ((S(a)(S(a)))(Sa) = ((SS)(aa))(Sa) \subseteq (Sa^2)S \subseteq \Sigma(Sa^2)S.$$  

This shows that $S$ is intra-regular. □

**Theorem 3.8.** Let $S$ be a pure left invertible LA-semihyperring with a pure left identity $e$. Then the following conditions are equivalent:

(i) $S$ is intra-regular;

(ii) $L \cap R \subseteq \Sigma LR$, where $L$ and $R$ are any left and right hyperideals of $S$, respectively.
Proof. (i) ⇒ (ii) Assume that $S$ is intra-regular. Let $L$ be a left hyperideal and $R$ be a right hyperideal of $S$, and let $a \in L \cap R$. Then, by using left invertive law and Lemma 2.3, we have
\[ a \in \Sigma(Sa^2)S = \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(S(Sa)a) \subseteq \Sigma(S(SL))R \subseteq \Sigma LR. \]

Hence, $L \cap R \subseteq \Sigma LR$.

(ii) ⇒ (i) Assume that (ii) holds. Let $a \in S$. Since $S$ is a pure left invertible, there exists $x \in S$ such that $e = xa$. By Lemma 2.12, $a^2S$ is both a left and a right hyperideal of $S$ such that $a^2 \subseteq a^2S$. Then, by using left interive law, Lemma 2.3 and given assumption, we have
\[ a^2 \subseteq a^2S \cap a^2S \subseteq \Sigma(a^2S)(a^2S) = \Sigma a^2((a^2S)S) = \Sigma a^2((SS)a^2) = \Sigma(aa)(S a^2) = \Sigma(S a^2)a. \]

Now, by using left invertive law and Remark 1, we have
\[ a = ea = (xa)a = (aa)x \subseteq (\Sigma(Sa^2)a)a)x \subseteq \Sigma((Sa^2)a)x = \Sigma(Sa^2)a \subseteq \Sigma(S a^2)S. \]

Therefore, $S$ is intra-regular. \[\Box\]

Theorem 3.9. Let $S$ be a pure left invertible LA-semihyperring with a pure left identity $e$. Then the following statements are equivalent:

(i) $S$ is intra-regular;

(ii) $L \cap R = \Sigma LR$, for every left hyperideal $L$ and every right hyperideal $R$ of $S$.

Proof. (i) ⇒ (ii) Assume that $S$ is intra-regular. Let $L$ and $R$ be a left hyperideal and a right hyperideal of $S$, respectively. It is easy to see that $\Sigma LR \subseteq L \cap R$. On the other hand, let $a \in L \cap R$. Then, $a \in \Sigma(Sa^2)S$. By using left invertive law, paramedial law and Lemma 2.3, we have
\[ a \in \Sigma(Sa^2)S = \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(S(Sa)a) = \Sigma((eS)(Sa)a) = \Sigma((eS)(Sa)a) \subseteq \Sigma((RS)(Sa)L \subseteq \Sigma LR. \]

Hence, $L \cap R \subseteq \Sigma LR$. Therefore, $L \cap R = \Sigma LR$.

(ii) ⇒ (i) This proof is similar to the proof of (ii) ⇒ (i) in Theorem 3.8, because $a^2S$ is both a left hyperideal and a right hyperideal of $S$. \[\Box\]

Theorem 3.10. Let $S$ be a pure left invertible LA-semihyperring with a pure left identity $e$ such that $(xe)S \subseteq xS$ for all $x \in S$. Then the following statements are equivalent:

(i) $S$ is intra-regular;

(ii) $G \cap I = (GI)G$, for every generalized bi-hyperideal $G$ and every hyperideal $I$ of $S$;

(iii) $B \cap I = (BI)B$, for every bi-hyperideal $B$ and every hyperideal $I$ of $S$;

(iv) $Q \cap I = (QI)Q$, for every quasi-hyperideal $Q$ and every hyperideal $I$ of $S$.

Proof. (i) ⇒ (ii) Assume that $S$ is intra-regular. Let $G$ be a generalized bi-hyperideal and $I$ be a hyperideal of $S$, and let $a \in G \cap I$. Then, $a \in \Sigma(Sa^2)S$. Now, by using left invertive law and Lemma 2.3, we have
\[ a \in \Sigma(Sa^2)S = \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(S(Sa)a). \]
Consider,

\[ S(Sa) \subseteq S(S(\Sigma(Sa^2)S)) \subseteq \Sigma S((Sa^2)(S \Sigma S)) = \Sigma S((Sa^2)(S \Sigma S)) \]
\[ = \Sigma(Sa^2)(S \Sigma S) \subseteq \Sigma(S(aa))S = \Sigma(a(Sa))S \]
\[ = \Sigma(S(Sa))a = (\Sigma S(Sa))a \subseteq S(a). \]

Then, by using (3.1), medial law, Lemma 2.3 and Lemma 2.14, we have

\[ S(Sa) \subseteq (\Sigma S(Sa))a \subseteq (\Sigma S(a))a = (S(a))(ea) = (S(e)(aa) = a((S(a))a) \subseteq a(S(a)) \subseteq S(Sa). \]

It follows that \( S(Sa) = a(Sa). \) Thus, \( a \in \Sigma(S(Sa))a = \Sigma(a(Sa))a = (a(Sa))a \subseteq (G(SI))G \subseteq (GI)G. \)

Hence, \( G \cap I \subseteq (GI)G. \) On the other hand, \( (GI)G \subseteq (SI)S \subseteq I \) and \( (GI)G \subseteq (GS)G \subseteq G, \) that is, \( (GI)G \subseteq G \cap I. \) Therefore, \( G \cap I = (GI)G. \)

(ii) \( \Rightarrow \) (iii) Since every bi-hyperideal is a generalized bi-hyperideal of \( S, \) it follows that (iii) holds.

(iii) \( \Rightarrow \) (iv) By Lemma 2.11, we have that every quasi-hyperideal of \( S \) is a bi-hyperideal. Hence, (iv) holds.

(iv) \( \Rightarrow \) (i) Let \( L \) be a left hyperideal and \( R \) be a right hyperideal of \( S. \) By Lemma 2.8 and Lemma 2.9, we have that \( R \) is a hyperideal and \( L \) is a quasi-hyperideal of \( S, \) respectively. By assumption, \( L \cap R = (LR)L \subseteq (SR)R \subseteq RL \subseteq \Sigma RL. \) On the other hand, \( \Sigma RL \subseteq L \cap R. \) Therefore, \( L \cap R = \Sigma R. \) By Theorem 3.9, we have that \( S \) is intra-regular.

**Theorem 3.11.** Let \( S \) be a pure left invertible LA-semihyperring with a pure left identity \( e \) such that \( (xe)S \subseteq xS \) for all \( x \in S. \) Then the following statements are equivalent:

(i) \( S \) is intra-regular;

(ii) \( R \cap G \subseteq \Sigma GR, \) for every generalized bi-hyperideal \( G \) and every right hyperideal \( R \) of \( S; \)

(iii) \( R \cap B \subseteq \Sigma BR, \) for every bi-hyperideal \( B \) and every right hyperideal \( R \) of \( S; \)

(iv) \( R \cap Q \subseteq \Sigma QR, \) for every quasi-hyperideal \( Q \) and every right hyperideal \( R \) of \( S. \)

**Proof.** (i) \( \Rightarrow \) (ii) Assume that \( S \) is intra-regular. Let \( R \) be a right hyperideal and \( G \) be a generalized bi-hyperideal of \( S, \) and let \( a \in R \cap G. \) Then, \( a \in \Sigma(Sa^2)S. \) Since \( S(Sa) \subseteq S(a), \) left invertive law, medial law and Lemma 2.3, we obtain that

\[ a \in \Sigma(Sa^2)S = \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(Sa)a \subseteq \Sigma(Sa)a \]
\[ = \Sigma((ae)a) = (S(e)(aa) = a((S(e)a) = a((S(a))a) \subseteq \Sigma G((RS)S) \subseteq \Sigma G(R). \]

Hence, \( R \cap G \subseteq \Sigma GR. \)

(ii) \( \Rightarrow \) (iii) Since every bi-hyperideal is a generalized bi-hyperideal of \( S, \) it follows that (iii) holds.

(iii) \( \Rightarrow \) (iv) By Lemma 2.11, we have that every quasi-hyperideal of \( S \) is a bi-hyperideal. Hence, (iv) holds.

(iv) \( \Rightarrow \) (i) Let \( L \) be a left hyperideal and \( R \) be a right hyperideal of \( S. \) By Lemma 2.9, \( L \) is a quasi-hyperideal of \( S. \) By assumption, \( L \cap R \subseteq \Sigma LR. \) Therefore, \( S \) is intra-regular by Theorem 3.8.

**Theorem 3.12.** Let \( S \) be a pure left invertible LA-semihyperring with a pure left identity \( e \) such that \( (xe)S \subseteq xS \) for all \( x \in S. \) Then the following conditions are equivalent:

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(i) \( S \) is intra-regular;
(ii) \( R \cap G \subseteq \Sigma R G \), for every generalized bi-hyperideal \( G \) and every right hyperideal \( R \) of \( S \);
(iii) \( R \cap B \subseteq \Sigma R B \), for every bi-hyperideal \( B \) and every right hyperideal \( R \) of \( S \);
(iv) \( R \cap Q \subseteq \Sigma R Q \), for every quasi-hyperideal \( Q \) and every right hyperideal \( R \) of \( S \).

Proof. (i) \( \Rightarrow \) (ii) Assume that \( S \) is intra-regular. Let \( G \) be a generalized bi-hyperideal and \( R \) be a right hyperideal of \( S \). Let \( a \in R \cap G \). Then, \( a \in \Sigma (S a^2)S \). Thus, by using left invertive law and Lemma 2.3, we have \( a \in \Sigma (S a^2)S = \Sigma (S(aa))S = \Sigma (a(aS))S = \Sigma (S(Sa))a \). Since \( S(Sa) = a(Sa) \), we have

\[ a \in \Sigma (S(Sa))a = \Sigma (a(Sa))a \subseteq \Sigma (RS)G \subseteq \Sigma RG. \]

This implies that \( R \cap G \subseteq \Sigma R G \).

(ii) \( \Rightarrow \) (iii) Since every bi-hyperideal is a generalized bi-hyperideal of \( S \), it turns out that (iii) holds.

(iii) \( \Rightarrow \) (iv) By Lemma 2.11, we have that every quasi-hyperideal of \( S \) is a bi-hyperideal. So, (iv) holds.

(iv) \( \Rightarrow \) (v) Let \( L \) and \( R \) be a left hyperideal and a right hyperideal of \( S \), respectively. By Lemma 2.9, \( L \) is also a quasi-hyperideal of \( S \). By hypothesis, \( L \cap R \subseteq \Sigma RL \). Otherwise, \( \Sigma RL \subseteq L \cap R \). Hence, \( L \cap R = \Sigma RL \). Therefore, \( S \) is intra-regular by Theorem 3.9.

Theorem 3.13. Let \( S \) be a pure left invertible LA-semihyperring with a pure left identity \( e \) such that \( xS \subseteq xS \) for all \( x \in S \). Then the following statements are equivalent:

(i) \( S \) is intra-regular;
(ii) \( L \cap G \subseteq \Sigma LG \), for every generalized bi-hyperideal \( G \) and every left hyperideal \( L \) of \( S \);
(iii) \( L \cap B \subseteq \Sigma LB \), for every bi-hyperideal \( B \) and every left hyperideal \( L \) of \( S \);
(iv) \( L \cap Q \subseteq \Sigma LQ \), for every quasi-hyperideal \( Q \) and every left hyperideal \( L \) of \( S \).

Proof. (i) \( \Rightarrow \) (ii) Assume that \( S \) is intra-regular. Let \( L \) be a left hyperideal and \( G \) be a generalized bi-hyperideal of \( S \), and let \( a \in L \cap G \). Then, \( a \in \Sigma (S a^2)S \). Now, by using left invertive law and Lemma 2.3, we have

\[ a \in \Sigma (S a^2)S = \Sigma (S(aa))S = \Sigma (a(aS))S = \Sigma (S(Sa))a \subseteq \Sigma (S(SL))G \subseteq \Sigma LG. \]

This implies that \( L \cap G \subseteq \Sigma LG \).

(ii) \( \Rightarrow \) (iii) Since every bi-hyperideal is a generalized bi-hyperideal of \( S \), it follows that (iii) holds.

(iii) \( \Rightarrow \) (iv) By Lemma 2.11, we have that every quasi-hyperideal of \( S \) is a bi-hyperideal. Hence, (iv) holds.

(iv) \( \Rightarrow \) (i) Let \( L \) be a left hyperideal and \( R \) be a right hyperideal of \( S \). By Lemma 2.9, \( R \) is also a quasi-hyperideal of \( S \). By assumption, \( L \cap R \subseteq \Sigma LR \). Therefore, \( S \) is intra-regular by Theorem 3.8.

Theorem 3.14. Let \( S \) be a pure left invertible LA-semihyperring with a pure left identity \( e \) such that \( xS \subseteq xS \) for all \( x \in S \). Then the following statements are equivalent:

(i) \( S \) is intra-regular;
(ii) \( L \cap G \subseteq \Sigma GL \), for every generalized bi-hyperideal \( G \) and every left hyperideal \( L \) of \( S \);
(iii) \( L \cap B \subseteq \Sigma BL \), for every bi-hyperideal \( B \) and every left hyperideal \( L \) of \( S \);
(iv) \( L \cap Q \subseteq \Sigma QL \), for every quasi-hyperideal \( Q \) and every left hyperideal \( L \) of \( S \).
Hence, \( L \cap G \subseteq \Sigma GL \).

(ii) \( \Rightarrow \) (iii) Since every bi-hyperideal of \( S \) is a generalized bi-hyperideal, it follows that (iii) holds.

(iii) \( \Rightarrow \) (iv) The implication holds from Lemma 2.11.

(iv) \( \Rightarrow \) (i) Let \( L \) and \( R \) be a left hyperideal and a right hyperideal of \( S \), respectively. By Lemma 2.9, \( R \) is also a quasi-hyperideal of \( S \). By the given assumption, we have \( L \cap R \subseteq \Sigma RL \). On the other hand, \( \Sigma RL \subseteq L \cap R \). Therefore, \( L \cap R = \Sigma RL \). By Theorem 3.9, we obtain that \( S \) is intra-regular. \( \square \)

**Theorem 3.15.** Let \( S \) be a pure left invertible LA-semihyperring with a pure left identity \( e \) such that \((xe)S \subseteq xS \) for all \( x \in S \). Then the following conditions are equivalent:

(i) \( S \) is intra-regular;

(ii) \( L \cap G \cap R \subseteq \Sigma (LG)R \), for every generalized bi-hyperideal \( G \), every left hyperideal \( L \) and every right hyperideal \( R \) of \( S \);

(iii) \( L \cap B \cap R \subseteq \Sigma (LB)R \), for every bi-hyperideal \( B \), every left hyperideal \( L \) and every right hyperideal \( R \) of \( S \);

(iv) \( L \cap Q \cap R \subseteq \Sigma (LQ)R \), for every quasi-hyperideal \( Q \), every left hyperideal \( L \) and every right hyperideal \( R \) of \( S \).

**Proof.** (i) \( \Rightarrow \) (ii) Assume that \( S \) is intra-regular. Let \( G \) be a generalized bi-hyperideal, \( L \) be a left hyperideal and \( R \) be a right hyperideal of \( S \), and let \( a \in L \cap G \cap R \). Then, \( a \in \Sigma (S a^2)S \). We note that \( S(Sa) = a(Sa) \). By using left invertive law, medial law, paramedial law and Lemma 2.3, we have

\[
a \in \Sigma (S a^2)S = \Sigma (a(Sa))S = \Sigma (S(Sa))a = (S(e)a)(ea) = (S(aa)(ea)
\]

Thus, by using left invertive law, medial law, paramedial law and Lemma 2.3, we have

\[
a \in \Sigma (S a^2)S = \Sigma (a(Sa))S = \Sigma (S(Sa))a = (S(e)a)(ea) = \Sigma (aa)((ae)S) \subseteq (LR)(RS)S \subseteq (LR)R.
\]

Hence, \( L \cap G \cap R \subseteq \Sigma (LG)R \).

(ii) \( \Rightarrow \) (iii) Since every bi-hyperideal is a generalized bi-hyperideal of \( S \), it follows that (iii) holds.

(iii) \( \Rightarrow \) (iv) By Lemma 2.11, we have that every quasi-hyperideal of \( S \) is a bi-hyperideal. Hence, (iv) holds.

(iv) \( \Rightarrow \) (i) Let \( L \) be a left hyperideal and \( R \) be a right hyperideal of \( S \). By Lemma 2.9, \( L \) is a quasi-hyperideal of \( S \). By assumption, \( L \cap R = L \cap L \cap R \subseteq \Sigma (LL)R \subseteq \Sigma (SL)R \subseteq SLR \). By Theorem 3.8, we obtain that \( S \) is intra-regular. \( \square \)

**Theorem 3.16.** Let \( S \) be a pure left invertible LA-semihyperring with a pure left identity \( e \) such that \((xe)S \subseteq xS \) for all \( x \in S \). Then the following statements are equivalent:

(i) \( S \) is intra-regular;

(ii) \( L \cap G \cap R \subseteq \Sigma (RG)L \), for every generalized bi-hyperideal \( G \), every left hyperideal \( L \) and every right hyperideal \( R \) of \( S \);
(iii) \( L \cap B \cap R \subseteq \Sigma(RB)L \), for every bi-hyperideal \( B \), every left hyperideal \( L \) and every right hyperideal \( R \) of \( S \);

(iv) \( L \cap Q \cap R \subseteq \Sigma(RQ)L \), for every quasi-hyperideal \( Q \), every left hyperideal \( L \) and every right hyperideal \( R \) of \( S \).

**Proof.** (i) \( \Rightarrow \) (ii) Assume that \( S \) is intra-regular. Let \( G \) be a generalized bi-hyperideal, \( L \) be a left hyperideal and \( R \) be a right hyperideal of \( S \). Let \( a \in L \cap G \cap R \). Then, \( a \in \Sigma(Sa^2)S \). Since \( S(Sa) \subseteq (\Sigma S (Sa))a \subseteq Sa \) and by Lemma 2.14, we have \( S(Sa) \subseteq (\Sigma S (Sa))a \subseteq (\Sigma S a) \subseteq (S a)a = (Sa)a \). By the given assumption, left invertive law, medial law, paramedial law and Lemma 2.3, we have

\[ a \in (\Sigma(Sa^2))S = \Sigma(S(Sa))a = \Sigma((S(a)e)a) = \Sigma((ae)(aS)) = (\Sigma((RS)(S)G)L) \subseteq (\Sigma(RG)L). \]

This shows that, \( L \cap G \cap R \subseteq \Sigma(RG)L \).

(ii) \( \Rightarrow \) (iii) Since every bi-hyperideal of \( S \) is a generalized bi-hyperideal, which implies that (iii) holds.

(iii) \( \Rightarrow \) (iv) The proof follows from Lemma 2.11.

(iv) \( \Rightarrow \) (v) Let \( L \) be a left hyperideal and \( R \) be a right hyperideal of \( S \). Also, \( L \) is a quasi-hyperideal of \( S \) by Lemma 2.9. By assumption, we have that \( L \cap R = L \cap L \cap R \subseteq (\Sigma RL) \subseteq (\Sigma RS)L \subseteq \Sigma RL \).

Otherwise, \( \Sigma RL \subseteq L \cap R \). Hence, \( L \cap R = \Sigma RL \). Therefore, \( S \) is intra-regular by Theorem 3.9. \( \square \)

The following theorem, we can prove similarly.

**Theorem 3.17.** Let \( S \) be a pure left invertible LA-semihyperring with a pure left identity \( e \) such that \((xe)S \subseteq xS \) for all \( x \in S \). Then the following conditions are equivalent:

(i) \( S \) is intra-regular;

(ii) \( R \cap G \subseteq \Sigma(RG)R \), for every generalized bi-hyperideal \( G \) and every right hyperideal \( R \) of \( S \);

(iii) \( R \cap B \subseteq \Sigma(RB)R \), for every bi-hyperideal \( B \) every right hyperideal \( R \) of \( S \);

(iv) \( R \cap Q \subseteq \Sigma(RQ)R \), for every quasi-hyperideal \( Q \) and every right hyperideal \( R \) of \( S \).

4. Conclusions

In 2018, the concept of LA-semihyperrings was introduced by Nawaz et al. [33] as a generalization of LA-semirings. In Section 2, we have shown that some LA-semihyperring can be constructed from an ordered LA-semiring as shown in Lemma 2.6. This means that the LA-semihyperring is also a generalization of an ordered LA-semiring. In Section 3, we applied the concept of intra-regular LA-rings, appeared in [21], to define the concept of intra-regular LA-semihyperrings and discussed some of its properties. Finally, we characterized the class of intra-regular LA-semihyperrings by using (resp., left, right) hyperideals, quasi-hyperideals, bi-hyperideals and generalized bi-hyperideals of LA-semihyperrings were shown in Theorem 3.7 - Theorem 3.17. In our future study, we can consider the characterizations of the class of both regular and intra-regular LA-semihyperrings based on different types of hyperideals of LA-semihyperrings.
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Conflict of Interest

The author declares no conflict of interest.

References


For more questions regarding reference style, please refer to the Citing Medicine.

Supplementary (if necessary)

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